THE NEOTIA UNIVERSITY

WORK INSTRUCTION

1.0 EXPERIMENT NO: BS/PHP101/02

2.0 NAME OF EXPERIMENT: Rigidity Modulus

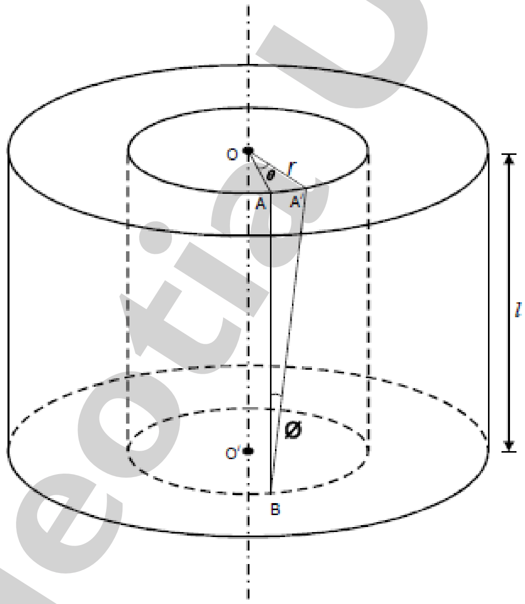
3.0 0BJECTIVE: determination of Rigidity Modulus of the material of an wire by dynamical method

4.0 THEORITICAL BACKGROUND:

Torsion of a wire:

We assume the shape of the wire as a cylinder of length l and radius a. Let it's one end be clamped and at the other end a torque is applied.As a result of the torque the cylinder is twisted through a certain angle.We imagine the cylinder in question is divided up into a number of co-axial discs, then due to this twist these discs will turn relative to one another about the axis of the cylinder and the amount of turning of a disc will depend on it's position from the fixed end. The disc which is far from the fixed end will turn more about the axis than the one nearer to the fixed end. To measure this turn, we consider an imaginary co-axial cylinder of radius r. A straight line AB is considered over the surface of this cylinder, parallel to the axis OO'. After twisting the line gets inclined (as A'B) to the original line. The angle between AB and A'B (i.e φ) is the angle of shear. AA' represents the amount of turn of the line OA which is on the surface of the cylinder. Arc length AA' is equal to rθ, where r is radius of the imaginary cylinder and $\angle AOA' = \theta$ (angle of twist) and arc length AA' is also l φ, Therefore rθ=l φ.

Hence $\varphi = \frac{r\theta}{l}$ ………………..(1)

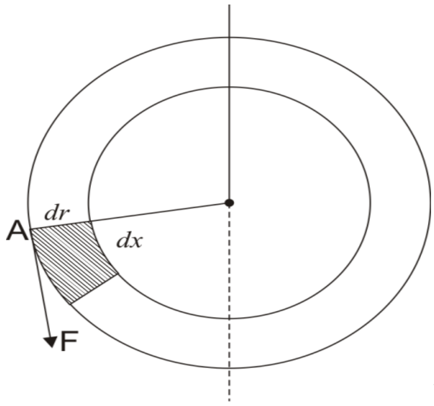


Now due to the elastic property of the material the relative turnung of the co-axial disc gives rise to a restoring torque which tends to prevent the twisting of the cylinder.

To calculate the restoring torque we proceed in the following way:

We imagine the cylinder to be divided into a number of co-axial cylindrical shells. We take into consideration one such shell of radius r and thickness dr. On the face of this shell at the position A we imagine an element of length dx, whose breadth is dr. Over this element let F be the tangential force that comes to play due to relative turning.

Tangential stress= $\frac{F}{dr\ dx}$



So rigidity modulus $\eta = \frac{\frac{F}{dr\ dx}}{\theta}$ (From 1)

$$= = \frac{F\,l}{dr\ dx\ r\ \varphi}$$

Hence $F = \frac{\eta\,\varphi\,r\,dr\,dx}{l}$ ………………(2)

The moment of this force about the axis of the cylinder is given by

$F.r = \frac{\eta\,\varphi r^2\,dr\,dx}{l}$

Therefore the restoring torque over the entire surface of the annulus is given by

$$\delta\Gamma = \frac{\eta\,\varphi r^2\,dr\ \sum dx}{l} = \frac{\eta\,\varphi r^2\,dr\ 2\pi r}{l} \quad (\sum dx = 2\pi r)$$

$$= \frac{2\pi\eta\,\varphi r^3\,dr}{l}$$

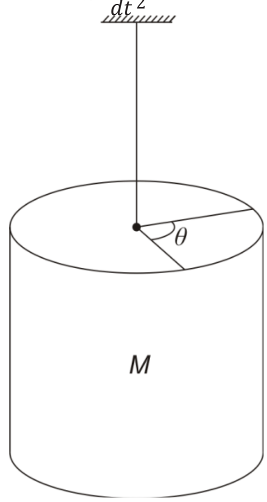
The total restoring torque that comes to play during the twisting of the entire cylinder of radius r is $\Gamma = \frac{2\pi\eta\varphi}{l}\int_0^r r^3 dr = \frac{2\pi\eta\varphi}{l}\frac{r^4}{4} = = \frac{\pi\eta\varphi\,r^4}{2l}$ ………………(3)

Tortional rigidity or tortional constant is restoring torque per unit twist

$C = \frac{\Gamma}{\varphi} = \frac{\pi\eta\,r^4}{2l}$ ………………………..(4)

Tortional Pendulum : In this pendulum a wire is fixed at one end and has a bob in the form of a solid cylinder of mass M at the bottom. Giving a slight rotation to M, the wire is twisted at the free end and the system is made to oscillate about he wire as axis.If θ be the angle of twist imparted to the wire then the deflecting couple is $I\frac{d^2\theta}{dt^2}$. I belng the moment of inertia of the solid cylinder of mass M about the wire.The restoring torque must be $C\theta$, where C is the restoring torque per unit twist or tortional rigidity.

$I\frac{d^2\theta}{dt^2} = -C\,\theta \Rightarrow \frac{d^2\theta}{dt^2} = -\frac{C}{I}\theta \Rightarrow \frac{d^2\theta}{dt^2} + \frac{C}{I}\theta = 0 \Rightarrow \frac{d^2\theta}{dt^2} + \omega^2\theta = 0$, where $\omega^2 = \frac{C}{I}$



The motion is simple harmonic motion with time period $T = \frac{2\pi}{\omega} \Rightarrow T = 2\pi\sqrt{\frac{I}{C}}$

$I = \frac{MR^2}{2}$ and $C = \frac{\pi\eta r^4}{2l}$ $\Rightarrow T = 2\pi\sqrt{\frac{MR^2/2}{\frac{\pi\eta r^4}{2l}}}$ $\Rightarrow T^2 = 4\pi^2 \frac{MR^2}{2}\frac{2l}{\pi\eta r^4}$

Therefore $\eta = 4\pi \frac{MR^2 l}{T^2 r^4}$ ....(5)

What is moment of inertia?

Let us consider a rigid body rotating about the axis AB with uniform angular velocity ω. We imagine that the body is made up of a large number of particles of masses $m_1, m_2, m_3, \ldots,$ situated at distances r_1, r_2, r_3, etc respectively from the axis of rotation. The angular velocity of different particles is same (ω) but their linear velocities are different due to their different linear positions with respect to the axis. Hence the kinetic energy of the rigid body

$E = \frac{1}{2} m_1 \omega^2 r_1^2 + \frac{1}{2} m_2 \omega^2 r_2^2 + \frac{1}{2} m_3 \omega^2 r_3^2 + \ldots\ldots\ldots\ldots\ldots\ldots$

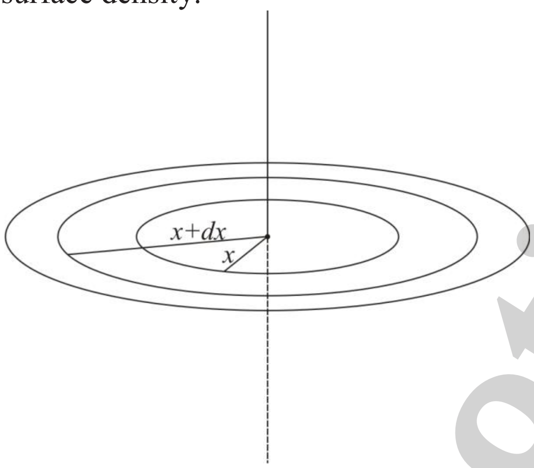
$= \frac{1}{2}\omega^2(m_1 r_1^2 + \frac{1}{2} m_2 r_2^2 + \frac{1}{2} m_3 r_3^2 + \ldots\ldots\ldots\ldots\ldots\ldots)$

$= \frac{1}{2} I\omega^2$

Where $I = \sum mr^2$ = moment of inertia of the body about the axis in question

<u>Moment of inertia of a cylinder</u>

To find the moment of inertia of a cylinder, we will find out the moment of inertia of a disc about an axis passing through its centre and perpendicular to its plane. In the diagram the moment of inertia of a ring whose inner radius is x and outer radius is x+dx is $\sigma[\pi(x+dx)^2 - \pi x^2]\, x^2 = 2\pi x\, dx\, \sigma x^2$, where σ is the surface density.



Moment of inertia of the disc $= \int_0^R 2\pi\sigma x^3 dx = 2\pi\sigma \frac{R^4}{4} = m\frac{R^2}{2}$, where $m = \pi\sigma R^2$ =mass of the disc. Hence the moment of inertia of the whole cylinder can be obtained by summing the moment of inertia of all the discs which made the cylinder. $I = \sum m\frac{R^2}{2} = M\frac{R^2}{2}$

5.0 PRINCIPLE: The time period (T) with which the bob of a torsion pendulum oscillates, about the suspension wire as axis is given by

$$T = 2\pi\sqrt{\frac{I}{C}} \qquad \text{Or} \qquad C = \frac{4\pi^2 I}{T^2}$$

Where *I* is the moment of inertia of the bob of mass *M* and radius *R* of the torsion pendulum about the suspension wire as its own axis to be given by

$$I = \frac{1}{2} MR^2$$

And *C* represents the resting couple exerted by the suspension wire of length *l* for one radian twist at its free end and is given by, $C = \frac{\eta\pi r^4}{2l}$

Where (η) is the rigidity of modulus of the material of the wire, while *I* and *r* are respectively the length and radius of the suspension wire. Hence we can write:

$$\eta = \frac{8\pi I l}{T^2 r^4}$$

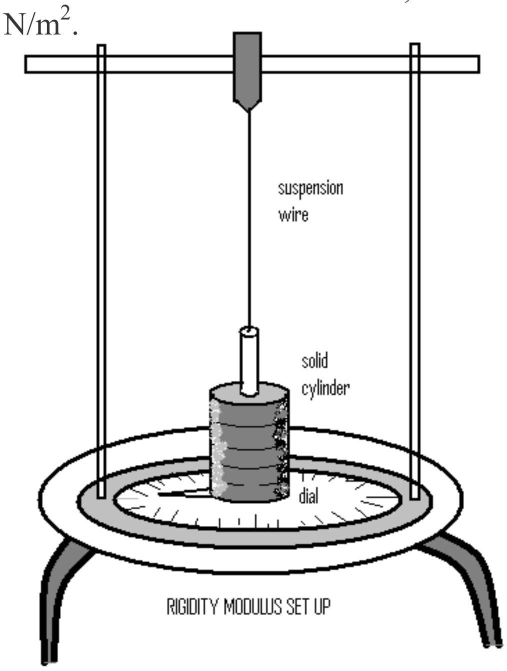
If we suspend a solid cylinder as the bob of the torsion pendulum, then by calculating *I* and measuring *l, r*, and *T* experimentally, we can determine the modulus of rigidity (η) of the wire from the following equation.

$$\eta = \frac{8\pi l}{T^2 r^4}\left(\frac{1}{2}\mathrm{MR}^2\right)$$

M and *R* are the mass and radius of the cylinder respectively. If *l*, *r* and *R* are measured in meters, *M* is measured in kg and *T* is measured in second then the unit of η will be N/m^2.

6.0 TOOLS/APPARATUS REQUIRED:

- Rigidity modulus set-up
- Measuring tape
- Slide calipers
- screw gauge
- Stop watch



7.0 PROCEDURE: (don't write the procedure in your Lab. Copy)

- Determine the vernier constant of the slide caliper (in Table – 1) and measure the radius (*R*) of the cylinder by the slide caliper as per instructions given in Table – 2.

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- Determine the least count of the screw gauge (in Table – 3) and measure the radius (*r*) of the suspension wire by the screw gauge as per instructions given in Table – 4.

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- Give a small twist to the suspension wire and determine the time taken for a certain number of torsional oscillations of the cylinder by a stop watch. Repeat this process for a number of times (say 5 – 6). Every time record the number of oscillations and its corresponding time in Table – 5. Plot a graph with number of oscillations (*n*) along the abscissa (x - axis) and time (*t*) taken for the oscillations along the vertices (y – axis). Determine the time period (*T*) from the graph as shown in Table – 5.

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- Measure length of suspension wire from the point of suspension to the point where the cylinder is attached and have the mass of the cylinder and then calculate the value of rigidity modulus of the wire as illustrated in Table – 6.

8.0 TABULATION:
Supplied, L= cm.
M = 1980 gm.
7.0 TABULATION:

TABLE-1
Vernier constant (*v.c.*) of the slide calipers

10 divisions (say *m*) of the vernier scale = 9 divisions (say n) of the main scale

Value of 1 smallest main scale division (l_1) (cm)	Value of 1 division of the vernier scale $\frac{n}{m}$	Vernier constant (*v.c.*) $(1-\frac{n}{m}) \times l_1$ (cm)

TABLE-2
Measurement of the radius (*R*) of the cylinder

No of obs.	Reading of the		Total reading (cm) $m_s + v_s \times v.c.$	Mean diameter *D* (cm)	Instrumental error (cm)	Correct *D* (cm)	Radius $R = D/2$ (cm)
	Main scale m_s (cm)	Vernier no. v_s					
1							
2							
3							

TABLE-3
Least count (*l.c.*) of the screw gauge

Pitch of the screw gauge is defined as the distance travelled by the circular scale for a complete rotation.

Pitch of the screw gauge *p* (cm)	No. of division of the circular scale *n*	Least count (*l.c.*) = *p*/*n* (cm)

TABLE-4

Measurement of radius (*r*) of the suspension wire by screw gauge

No of obs.	Reading of the		Total reading (cm) $m_s + c_s \times l.c.$	Mean diameter d (cm)	Instrumental error (cm)	Correct d (cm)	Radius $r = d/2$ (cm)
	Main scale m_s (cm)	Circular scale no. c_s					
1							
2							
3							

TABLE-5

Determine the time period (T) of the torsional oscillation of the cylinder

No of obs.	No of oscillation (n)	Total time taken for the oscillation (t) (sec.)	Value of an oscillation from graph (n_g)	Corresponding time for that oscillation from graph (t_g) (sec.)	Time Period (T) $= t_g/n_g$ (obtained from graph) (sec.)
1					
2					
3					
4					
5					
6					

Graph and Result

From the data of the Table – 5 plot a graph with number of oscillation (n) along the abscissa (x - axis) and time (t) taken for the oscillations along the vertices (y – axis). Determine the time period (T) from the graph. Therefrom calculate the value of the modulus of rigidity of the wire as illustrated in Table -6.

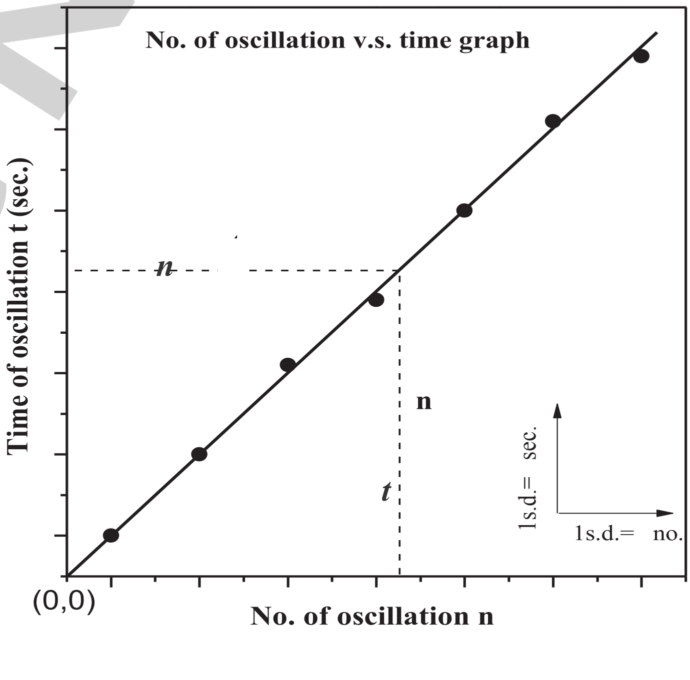

TABLE-6
Determination of Modulus of Rigidity (η)

L = m.
M = kg.

Value of R from TABLE - 2 (m)	Value of r from TABLE - 4 (m)	Value of T from TABLE - 5 (sec.)	Moment of inertia $I = \frac{1}{2} MR^2$ (kg-m^2)	Value of $\eta = \frac{8\pi L}{T^2 r^4} I$ (N/ m^2)

Ideal value of η (in SI unit)**:** (7 ~ 9) ×10^{10} N/m^2

9.0 COMPUTATION OF PERCENTAGE ERROR:

We have the formula for the

Rigidity modulus is: $\eta = \frac{8\pi L}{T^2 r^4}(\frac{1}{2} MR^2)$

Taking 'ln' in both sides we get: $\ln \eta = \ln L + \ln M + 2\ln R - 2\ln T - 4\ln r$

Differentiating both sides, we get for maximum proportional error, $\frac{\partial\eta}{\eta} = \frac{\partial L}{L} + \frac{2\partial R}{R} + \frac{2\partial T}{T} + \frac{4\partial r}{r}$

(For maximum proportional error the negative signs will be converted into positive signs. As M is given it will be eliminated from the equation after differentiation.)

δL= error in measuring L
= 2 × smallest division of the scale used (because probability of making error is in both sides)
δR= error in measuring R
= *v.c.* of the slide calipers (because probability of making error is in one side)
δT= error in measuring T
= one smallest division of stop watch (because probability of making error is in one side)
δr= error in measuring r
= *l.c.* of the screw gauge (because probability of making error is in one side)

By putting suitable values of L, R, T, and r calculate proportional error $\frac{\partial\eta}{\eta}$

Hence, the percentage error is: ($\frac{\partial\eta}{\eta}$ x 100) %

DISCUSSION:

You have to write all the difficulties you faced during the experiment and their remedies. Also you have to think and to mention some way out that one should adopt during the practical for getting a better result.

References

1) GENERAL PROPERTIES OF MATTER – Sengupta & Chatterjee
2) MECHANICS – Gupta & Kumar
3) MECHANICS - Spiegel