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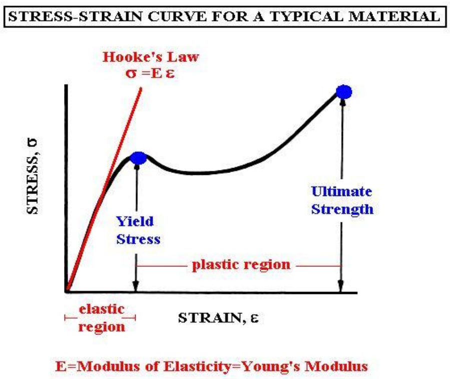
WORK INSTRUCTION

1.0 EXPERIMENT NO: BS/PHP101/01
2.0 NAME OF EXPERIMENT: Young's modulus
3.0 0BJECTIVE: Determination of Young's modulus by flexure method and calculation of bending moment and force at a point on the beam
4.0 TEORITICAL BACKGROUND:

Stress and Strain:

Stress is the restoring force caused due to the deformation divided by the area to which the force is applied; and strain is the ratio of the change caused by the stress to the original state of the object.

It states that within elastic limit, stress is proportional to the strain.
Thus $\frac{stress}{strain}$ is constant. The value of this constant is called modulus of elasticity.
The validity of Hooke's law is discussed below.



The first part of the curve i,e. in elastic region shows that the strain is proportional to the load per unit area or stress. Up to the point A Hooke's law holds and also up to end of first linear region we see when we unload the wire, the stretch wire shortens until its length is same as it was before the weight was given. This means that the wire is perfectly elastic up to A. When we go beyond this range we exceed what is known as elastic limit. Beyond the yield point the plastic region is started where a great extension start to act for any material.
There are different moduli of elasticity depending on the nature of the strain. We have from the following moduli of elasticity: Young's modulus, Bulk modulus, Rigidity modulus and Axial modulus. The moduli of electricity is called elastic constant of a substance.

Young's modulus,

$$Y = \frac{applied\ \ load\ \ per\ \ unit\ \ cross-section}{increase\ \ in\ length\ per\ \ unit\ \ length}$$

$$Y = \frac{longitudinal\ stress}{longitudinal\ strain}$$

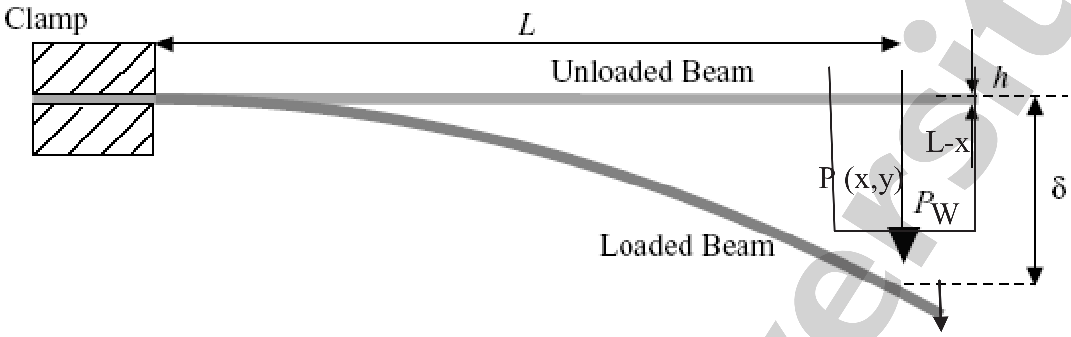
If F be the applied tensile force which causes a body of length l and cross-section A to elongate by x in the direction of force, the longitudinal stress is equal to F/A and the longitudinal strain is x/l. Hence, we express Young's modulus as

$$Y = \frac{F/A}{x/l} = \frac{Fl}{Ax}$$

In absolute C.G.S. units, F/A is expressed as in dynes/cm^2, while x/l is a pure number. Hence Young's modulus is expressed in dynes/cm^2.

Cantilever:

A cantilever is a uniform beam fixed horizontally at one end and loaded at the other end.



We consider a light cantilever fixed at clamp end and loaded at the free end by a concentrated weight *W*. Let P (x,y) be any point on the beam and *L* be the length of the beam.

The applied bending moment due to the load W about the point P which is at a distance x from the fixed end = *W (L-x)*.

Hence the bending moment equation is given by
$\frac{YAk^2}{R} = W(L-x)$ [Since the cantilever is light weight and is supposed to be weight less]
Where *A* is the sectional area of the beam and *k* is the radius of gyration.
The left-hand side of above equation represents the internal bending moment.
or, $YAk^2\frac{d^2y}{dx^2} = W(L-x)$ [when the bending is very small the curvature *1/R* at P may be taken to be equal to $\frac{d^2y}{dx^2}$.]

Note:
Radius of Gyration: We know, moment of inertia $I = \sum mr^2 = Mk^2$ *(say), where M denote the mass of the whole body and k is the dimension of length. k is called radius of gyration of the body about the axis. If we consider the whole mass of the body to be concentrated at the single point such that the moment of inertia of this concentrated point mass is same as the moment of inertia of the whole body about the axis, the distance of the single point from the axis is called the radius of gyration of the body about the axis.*
$\frac{YAk^2}{R}$ *is called the moment of internal resisting couple or internal bending moment.*
Yak^2 *is called flexural rigidity of the beam and is defined as the external bending moment required producing unit radius of curvature. It measures the resistance of the beam to bending.*

On integration, we have,
$YAk^2\frac{dy}{dx} = W(Lx - \frac{x^2}{2}) + C_1$, C_1 being the integration constant.
$\frac{dy}{dx} = 0$, when $x = 0$, because at the fixed end the bar remains horizontal.
So, $C_1 = 0$.
We have, $YAk^2\frac{dy}{dx} = W(Lx - \frac{x^2}{2})$.
On further integration, we have
$YAk^2y = W(\frac{Lx^2}{2} - \frac{x^3}{6}) + C_2$, C_2 being another integration constant.
Again, $y = 0$, when $x = 0$, and therefore, $C_2 = 0$.
So, $YAk^2y = W(\frac{Lx^2}{2} - \frac{x^3}{6})$
This gives the expression for the depression of the beam at any point.
Now at the point of loading, $x = L$ and let l be the depression of the beam. We have, then

$$YAk^2l = W\left(\frac{L^3}{2} - \frac{L^3}{6}\right) = \frac{WL^3}{3}$$

Or, $l = \frac{WL^3}{3YAk^2}$

If the beam be of rectangular cross-section of breadth a and thickness b, we have

$Ak^2 = \frac{ab^3}{12}$. Hence, $l = \frac{WL^3}{3Y\frac{ab^3}{12}} = \frac{4WL^3}{Yab^3}$

Since, W = Weight = mg, We have $Y = \frac{gL^3 m}{4bd^3 l}$

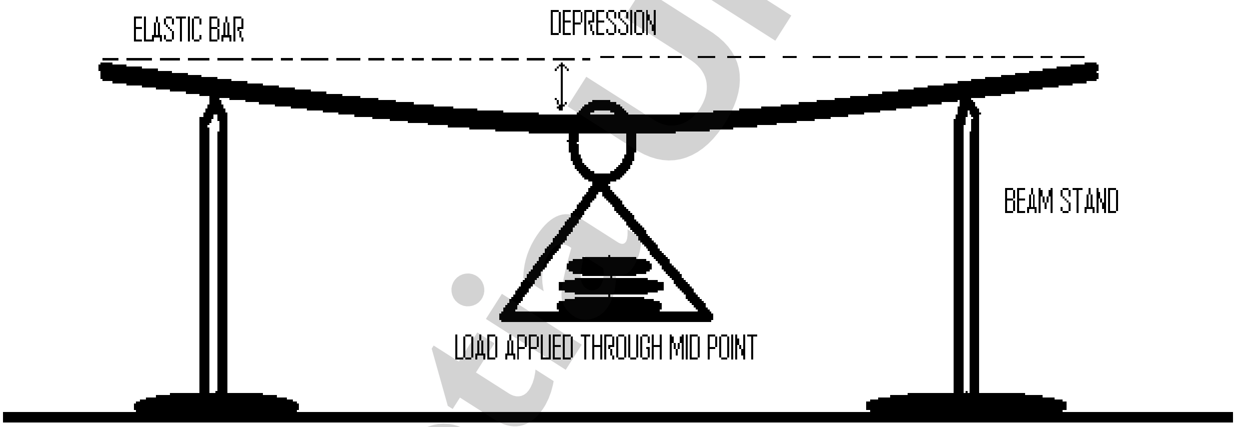
5.0 PRINCIPLE: If a light bar of breadth *b* and depth *d* is placed horizontally on two knife edges separated by a distance *L*, and a load of mass *m* , applied to the midpoint of the bar, produce a depression l of the bar, then young's modulus *Y* of the material of the bar is given by

$$Y = \frac{gL^3 m}{4bd^3 l} \qquad (1)$$

Where, g is the acceleration due to gravity. The bending moment is given by -

$$M = \frac{gL^3 m}{24\, l^2 + 6L^2} \qquad (2)$$

The equations (1) and (2) are the working formula of the experiment.



YOUNG'S MODULUS SET-UP

6.0 TOOLS/APPARATUS REQUIRED:

- Bending bar
- Traveling microscope
- Hanger
- Two stand with knife edge
- Slotted weight
- Screw gauge
- Slide caliper
- Meter Scale

7.0 PROCEDURE: (Don't write the procedure in your Lab. Copy)

- Measure the length by a meter scale between the two points of the given bar that have to be placed on the knife edge. Mark these two points and the midpoint between them.

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- Determine the vernier constant of travelling microscope of travelling microscope, the vernier constant of the slide calipers, and the least count of the screw gauge, and as illustrated in respectively Table – 1, Table -2, and Table -3.

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- Measure the breadth of the bar by the slide caliper and depth of the bar by the screw gauge. Note down the readings for the breadth in Table – 4 and those for the depth in Table – 5.

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- Mount the bar on the supporting stand with the two marked points placed on the knife edge and hang a hanger that has a vertical pointer from the marked midpoint of the bar. Observe and focus the tip of the pointer of the hanger through the eyepiece of the microscope. Adjust the eyepiece vertically so that the image of the pointer touches the horizontal cross-wire of the eye-piece without any load. Note down the readings of the main scale and vernier scale attached to the vertical arm of the microscope. These readings give you the position of the pointer without any load on the hanger.

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- Place a load of 0.5 kg on the hanger. This will produce a depression on the bar. Again focus the tip of the pointer of the hanger through the eyepiece of the microscope and adjust the eyepiece vertically so that the image of the pointer touches the horizontal cross-wire of the eye-piece. Note down the readings of the main scale and vernier scale attached to the vertical arm of the microscope. In Table – 1 these readings give you the depressed point (x) of the pointer placed on the bar corresponding to the load of 0.5 kg.

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- Increase gradually the load by 0.5 kg each time and repeat the above step to determine the depressed point of the rod until load on the hanger becomes 2.5 kg.

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- Now start to decrease the load from the hanger. For this purpose remove the load of 0.5 kg each time and record the data until the load on the hanger becomes 0. During decreasing the load take the readings of the main scale and vernier scale attached to the vertical arm of the microscope for each time. In this way you will get the elevated position (y) of the pointer placed on the bar while the load is decreasing.

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- Calculate the average of x and y for each load. It will give you various values of D corresponding to different load.

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- From the values of D determine the depression of the rod corresponding to various loads as shown in the Table – 6.

8.0 TABULATION:

Length between the two points of the given bar placed on the knife edge (L) =

TABLE – 1A

Vernier constant ($vc1$) of travelling microscope

50 divisions (say m) of the vernier scale=49 divisions (say n) of the main scale

Value of 1 smallest main scale division (l_1) (cm)	Value of 1 division of the vernier scale $\frac{n}{m}$	Vernier constant ($vc1$) $=(1 - \frac{n}{m}) \times l_1$ (cm)

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TABLE-2
Vernier constant ($vc2$) of the slide calipers

10 divisions(say m) of the vernier scale = 9 divisions (say n) of the main scale

Value of 1 smallest main scale division (l_1) (cm)	Value of 1 division of the vernier scale $\frac{n}{m}$	Vernier constant ($vc2$) $=(1 - \frac{n}{m}) \times l_1$ (cm)

TABLE-3
Least count ($l.c.$) of the screw gauge

Pitch of the screw gauge is defined as the distance travelled by the circular scale for a complete rotation.

Pitch of the screw gauge p (cm)	No. of division of the circular scale n	Least count ($l.c.$) $= p/n$ (cm)

TABLE-4
Measurement of breadth (b) of the bar by slide calipers

No of obs.	Reading of the		Total reading $= m_s + v_s \times vc2$ (cm)	Mean b (cm)	Instrumental error (cm)	Correct b (cm)
	Main scale m_s (cm)	Vernier no. v_s				
1						
2						
3						

TABLE-5
Measurement of depth (d) by screw gauge

No of obs.	Reading of the		Total reading $= m_s + c_s \times l.c.$ (cm)	Mean d (cm)	Instrumental error (cm)	Correct d (cm)
	Main scale m_s (cm)	Circular scale no. c_s				
1						
2						
3						

TABLE-6

Determination of depression for various loads

No of obs.	Load m (kg.)	Microscope reading for						Avg. Microscope reading $\frac{(x+y)}{2}$ (cm)	Depression l (cm)
		Increasing load (x)			Decreasing load (y)				
		Main scale m_s (cm)	Vernier reading v_s	Total reading $= m_s + v_s \times vc1$ (cm)	Main scale m_s (cm)	Vernier reading v_s	Total reading $= m_s + v_s \times vc1$ (cm)		
1	0.0							D_0=	0.0
2	0.5							D_1=	$D_0 - D_1$=
3	1.0							D_2=	$D_0 - D_2$=
4	1.5							D_3=	$D_0 - D_3$=
5	2.0							D_4=	$D_0 - D_4$=
6	2.5							D_5=	$D_0 - D_5$=

Graph and Result

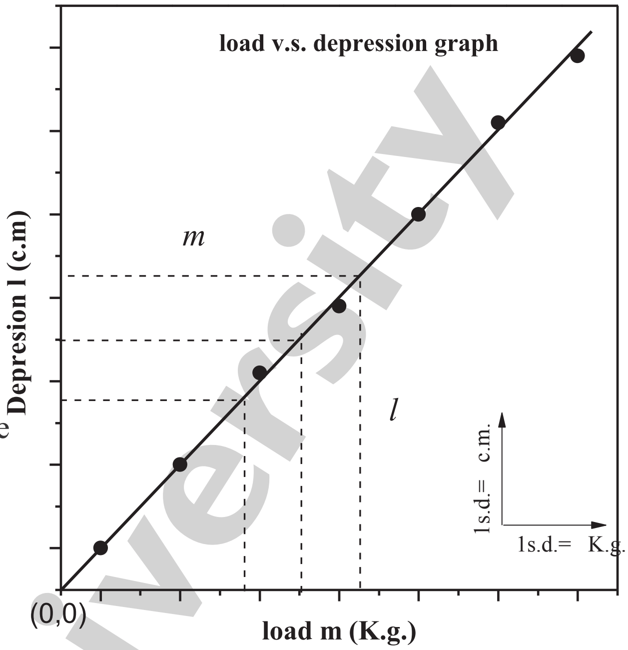
From the data of the Table – 6 plot a graph with load (m) along abscissa (x - axis) and depression (l) along vertices (y – axis). Determine the values of m and l from the graph. There from calculate the value of the Young's modulus and Bending Moment as illustrated in Table -7.

TABLE-7

Determination of Young's Modulus and Bending Moment

Value of g m/sec^2 (supplied)	Value of L (m)	Value of b (m)	Value of d (m)	Value of m from graph (kg.)	Value of l from graph (m)	Young's Modulus $Y = \frac{gL^3m}{4bd^3l}$ N/m^2	Bending Moment $M = \frac{gL^3m}{24\,l^2 + 6L^2}$ (J)	Mean Young's Modulus Y N/m^2	Mean Bending Moment M (J)

IDEAL VALUE OF *Y*: in SI unit 2×10^{11} N/m^2 and in cgs unit 2×10^{12} dyne/cm^2



9.0 COMPUTATION OF PERCENTAGE ERROR:

We have the formula for

Young's Modulus as: $Y = \dfrac{gL^3 m}{4bd^3 l}$

Taking ln in both sides we get,

lnY= ln g + 3 ln L + ln m – ln 4 – ln b – 3 ln d – ln l

Differential both sides we get for maximum

proportional error, $\dfrac{\partial Y}{Y} = \dfrac{\partial b}{b} + \dfrac{3\partial d}{d} + \dfrac{\partial l}{l}$

(For maximum proportional error the negative signs will be converted into positive signs. As L is given, g, m and 4 are constants; they will be eliminated from the equation after differentiation.)

Now,

δb= error in measuring b

= *v.c.* of the slide calipers (*vc*2) (because probability of making error is in one side)

δd= error in measuring d

= Least count (*l.c.*) of the screw gauge (because probability of making error is in one side)

δl= error in measuring l

= 2 × *v.c.* of the traveling microscope (*vc*1) (because probability of making error is both sides as we are subtracting two readings)

Putting suitable values calculate the value of proportional error.

Hence, the percentage error is ($\dfrac{\partial Y}{Y}$ × 100) % (Put the values and calculate the percentage error.

DISCUSSION:

You have to write all the difficulties you faced during the experiment and their remedies. Also you have to think and to mention some way out that one should adopt during the practical for getting a better result.

References

1) GENERAL PROPERTIES OF MATTER – Sengupta & Chattrjee
2) MECHANICS – Gupta & Kumar
3) MECHANICS - Spiegel
4) ADVANCED PRACTICAL PHYSICS- Ghosh & Majumdar