THE NEOTIA UNIVERSITY

ज्ञानम् आत्म प्रदीपाय

DEPARTMENT OF ROBOTICS & AUTOMATION

Inverse Kinematics

LAB MANUAL

THE NEOTIA UNIVERSITY
DEPARTMENT OF ROBOTICS & AUTOMATION

EXPERIMENT NO.: 1

NAME OF THE EXPERIMENT: Assignment on Introduction to Robot Configuration

OBJECTIVE: To study an introduction to Robot configuration.

THEORY:

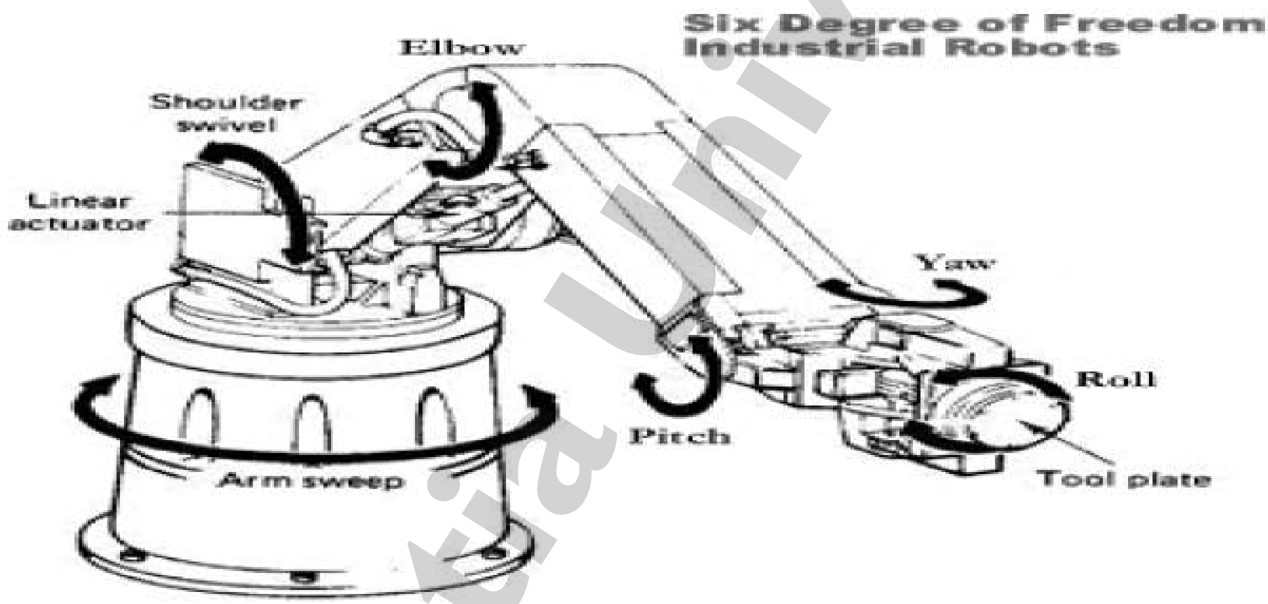
Robotics is the science of designing and building robots suitable for real-life applications in automated manufacturing and other non-manufacturing environments. Robot are the means of performing multifarious activities for man's welfare in the most planned and integrated manner, maintaining their own flexibility to do any work, effecting enhanced productivity, Guaranteeing quality, assuring reliability and ensuring safety to the workers. When the early man started settling in villages, they invented many innovative implements and left behind inscriptions to communicate many of their ideas.

To facilitate the manufacture of products, attempts were made to reduce human and animal labour, and to employ efficient machines run by exploiting other direct or converted natural energy sources. Meanwhile, the economic rule of demand and supply became operative. To produce more goods in a reasonably shorter period of time, the speed of production emerged as a factor of paramount importance. For the given five-M inputs (Man, machines, materials, money and motivation), more outputs at faster speed became imperative to raise the level of productivity. Gradually, the degree of mechanization in real life increased by employing more machines in place of direct labour. Higher heights of mechanization achieved every century, decade or year rendered the newer machines indispensable.

Programmable automation uses information technology and numerical engineering to provide coordination, machine control and communication through computers in the most effective way.af attempts to bridge the gap between consistency and flexibility.

An example of the Programmable automation technology is the robot. The robot is an essential component of CAM and CIM technologies. The name robot came from the Czechoslovakin word Robota which means a worker or a slave doing heavy work. The protoplasm of modern industrial robots is formed of hydraulics, pneumatics, electrical drives and silicon chips. Today's robots are therefore, to a great extent, as smart and intelligent as the robots conceived in fiction. Present-day industrial robots can work efficiently in both structured and unstructured environments. So, robots with their sensory capabilities and artificial intelligence (AI) are more advanced than the conventional and automated machines in all respects.

SPECIFICATIONS OF ROBOTS:



Accuracy:

How close does get the robot to the desired point? When the robot's program instructs the robot to move a desired point, it does not actually performed as per specified. The accuracy measures such as variance. That is the distance between the specified position that a robot is trying to achieve (programming point), and the actual x, y, and z resultant position of the robot end effecter.

Repeatability:

The ability of a robot to return to repeatedly to a given position. It is the ability of the robotic system or mechanism to repeat the same motion or achieve the same position. Repeatability is a measure of error or variability when repeatedly reaching for a single position. Repeatability is often smaller than accuracy.

Degree of Freedom (DOF):

Each joint or axis on the robot introduces a degree of freedom. Each DOF can be a slider, rotary, or other type of actuator. The number of DOF that a manipulator possesses thus is the number of independent ways in which a robot arm can move. Industrial robots typically have 5 or 6 degrees of freedom. 3 of the degrees of freedom allow positioning in 3D space (X, Y, Z), while the other 2 or 3 are used for orientation of the end effector (yaw, pitch and roll). 6 degrees of freedom are enough to allow the robot to reach all positions and orientations in 3D space. 5 DOF requires a restriction to 2D space, or else it limits orientations. 5 DOF robots are commonly used for handling tools such as arc welders.

Resolution:

The smallest increment of motion or distance that can be detected or controlled by the robotic control system. It is a function of encoder pulses per revolution and drive (e.g. reduction gear) ratio. And it is dependent on the distance between the tool centre point and the joint axis.

Reach:

The maximum horizontal distance from the center of the robot base to the end of its wrist.

Maximum Speed:

A robot moving at full extension with all joints moving simultaneously in complimentary directions at full speed. The maximum speed is the theoretical values which does not consider under loading condition.

Payload:

The maximum payload is the amount of weight carried by the robot manipulator at reduced speed while maintaining rated precision. Nominal payload is measured at maximum speed while maintaining rated precision. These ratings are highly dependent on the size and shape of the payload due to variation in inertia.

Envelope:

A three-dimensional shape that defines the boundaries that the robot manipulator can reach; also known as reach envelope.

Maximum envelope:

The envelope that encompasses the maximum designed movements of all robot parts, including the end effector, workpiece and attachments.

Restricted envelope:

Restricted envelope is that portion of the maximum envelope which a robot is restricted by limiting devices.

Operating envelope:

The restricted envelope that is used by the robot while performing its programmed motions.

BASIC CONFIGURATIONS OF INDUSTRIAL ROBOTS WITH THEIR APPLICATIONS:

Industrial robots come in a variety of shapes and sizes. They are capable of various arm manipulations and they possess different motion systems. This section discusses the various basic physical configurations of robots.

The following four basic Configurations can be combined in various ways to produce a variety of robotic combinations.

Cartesian Configuration:

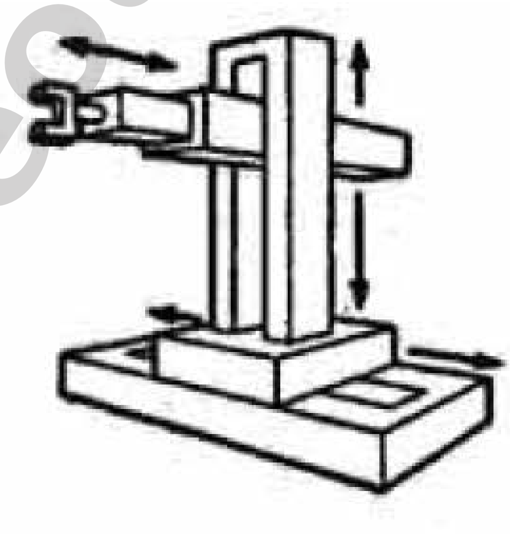


Fig: Cartesian Configuration

Cartesian robot is form by 3 prismatic joints, whose axes are coincident with the X, Y and Z planes.

In the Cartesian coordinate configuration shown in figure, the three orthogonal directions are X,Y and Z. X-coordinate axis may represent left and right motion; Y- coordinate axis may describe forward and backward motion; Z-coordinate axis may be used to represent up and down motions. Motions in any coordinate axis can be imparted independently of the other two. The manipulator can reach any point in a cubic volume of space. It allows three DOFs (x, y, z) in translation only.

Advantages:

1) 3 linear axes.
2) Easy to visualize, ability to do straight line insertions into furnaces.
3) Most rigid structure for given length.
4) Easy computation and programming.

Disadvantages:

1) Can only reach front of it.
2) Requires large floor space.
3) Axes hard to seal.

Applications:
Pick and Place operations, Assembly and Sub-Assembly (Mostly Straight), automated loading CNC Lathe and Milling operations, Nuclear Material handling, Welding etc.

Spherical Configuration:

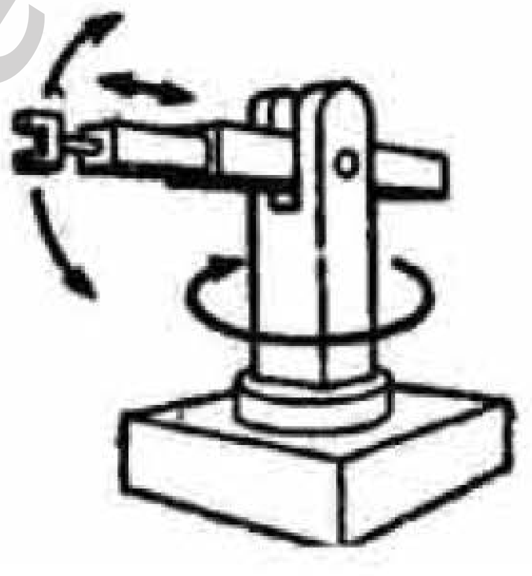


Fig: Spherical Configuration

In the Spherical coordinate configuration shown in figure, the robot has one linear and two angular motions. The linear motion, r corresponds to a radial in or out translation, the first angular motion corresponds to a base rotation, and second angular motion ,is one that rotates about an axis perpendicular to the vertical through the base and is sometimes termed as elbow rotation. The two rotations along with the in or out motion enable the robot to reach an specified point in the space bounded by an outer and inner hemisphere. Sometimes, the spherical coordinate system is referred to as polar coordinate system.

It is still in the research laboratory, the Spherical robot is actually a spherical shape robot, which has an internal driving source.

Advantages:
1) 1 linear + 2 rotating axes.
2) Large working envelops.
Disadvantages:
1) Can't reach around obstacles.
2) Low accuracy.
3) Complex coordinates more difficult to visualize, control, and program.

Applications:
Die Casting, Dip Coating, Forging, Glass Handling, Heat Treating, Injection Moulding, Machine Tool Handling, Material Transfer, Parts cleaning, Press Loading etc.

Cylindrical Configuration:

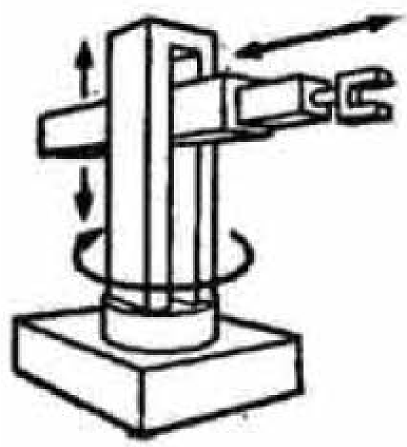


Fig: Cylindrical Configuration

Cylindrical robot is able to rotate along his main axes forming a cylindrical shape.

In the cylindrical coordinate configuration shown in figure, Consists of a vertical column, relative to which an arm assembly is moved up or down. The arm can be moved in or out relative to the column. A cylindrical robot has a two orthogonal prismatic axes of movement (horizontal and vertical) and one revolute axis, forming a cylindrical coordinate system. It is capable of higher horizontal plane speeds vs. cartesian systems due the revolute base. However horizontal, straight line motion is more complex to calculate and tends to be slower. The resolution of the positioning of the end effector is not constant, but depends on the degree of extension along the horizontal axis. If a monomast construction is used for the horizontal element, clearance behind the robot must be accounted for when retracted.

Advantages:
1) 2 linear axes +1 rotating.
2) Can reach all around itself.
3) Reach and height axes rigid.
4) Rotational axis easy to seal.
5) Relatively easy programming.

Disadvantages:
1) Can't reach above itself.
2) Base rotation axis as less rigid.
3) Linear axes are hard to seal.
4) Won't reach around obstacles.

Applications:
Assembly, Coating Applications, Conveyor Pallet Transfer, Die Casting, Forging Applications, Inspection Moulding, Investment Casting, Machine Loading and Unloading etc.

Jointed arm Configuration:

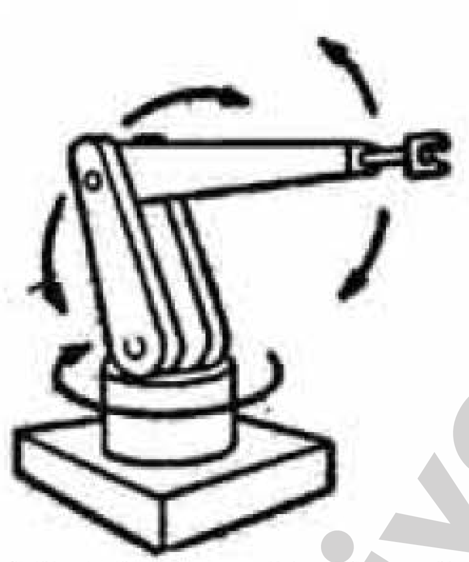


Fig: Jointed arm Configuration

Jointed arm Configuration robots are mechanic manipulator that looks like an arm with at least three rotary joints. The workspace of an articulated arm is complex, often a three- dimensional crescent. With all joints revolute, this type of robot requires the most complex kinematic calculations. An articulated configuration can most closely approximate an anthropomorphic, or human-arm motion, and thus offers a high degree of flexibility for accessing objects, devices or workstations within it's work envelope. Articulated robots may have two or more joints, with highly complex examples having as many as ten joints. A higher degree of flexibility comes at the cost of higher overall complexity, slower speed and higher cost. The resolution of the positioning of the end effector is not constant throughout the workspace.Positional repeatability can be more effected by gravity and load weight than other types because of the joints are oriented orithoginal to gravity.

Advantages:

1) All rotary joints allows for maximum capacity.
2) Any point in total volume can be reached.
3) All joints can be sealed from the environment.

Disadvantages:

1) Extremely difficult to visualize, control, and program.
2) Low accuracy.

Applications:

Assembly operations, Welding, Spray painting, Weld sealing etc.

SCARA:

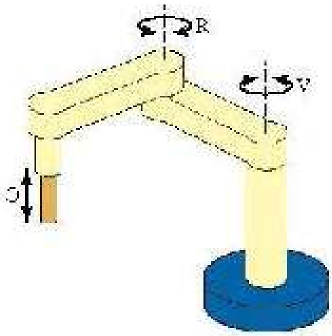


Fig: SCARA

SCARA stands for Selectively Compliant Assembly Robot Arm.

Similar to jointed-arm robot except that vertical axes are used for shoulder and elbow joints to be compliant in horizontal direction for vertical insertion tasks. It consists of two or more revolute joints and one prismatic, all of which operate parallel to gravity, easing the mechanical burden. As the name indicates, this configuration has been designed to offer variable compliance in horizontal directions, which can be an advantage in assembly tasks. The kinematics of this configuration are quite complex and the vertical component of movement is generally rather limited.Thus, it can reach around objects in the workspace, but not over them. The resolution of the positioning of the end effector is not constant throughout the workspace, but these robots do have a high degree of positional repeatability. They are generally faster and more expensive than cartesian systems.

Advantages:

1) 1 linear + 2 rotating axes.
2) Height axis is rigid.
3) High Speed.
4) Large work area for floor space.
5) Moderately easy to program.

Disadvantages:

1) Difficult to program off-line.
2) Highly complex arm.
3) 2 ways to reach point.
4) Limited Applications.

Applications:

Assembly operations, Pick and Place work etc.

CONCLUSION:

Hence, we have studied the Robot Configurations.

EXPERIMENT NO.: 2

NAME OF THE EXPERIMENT: Demonstration of Robot with 2 DOF, 3 DOF, 4 DOF etc.

OBJECTIVE: To Study of ROBOT with 2DOF, 3DOF & 4DOF.

THEORY:

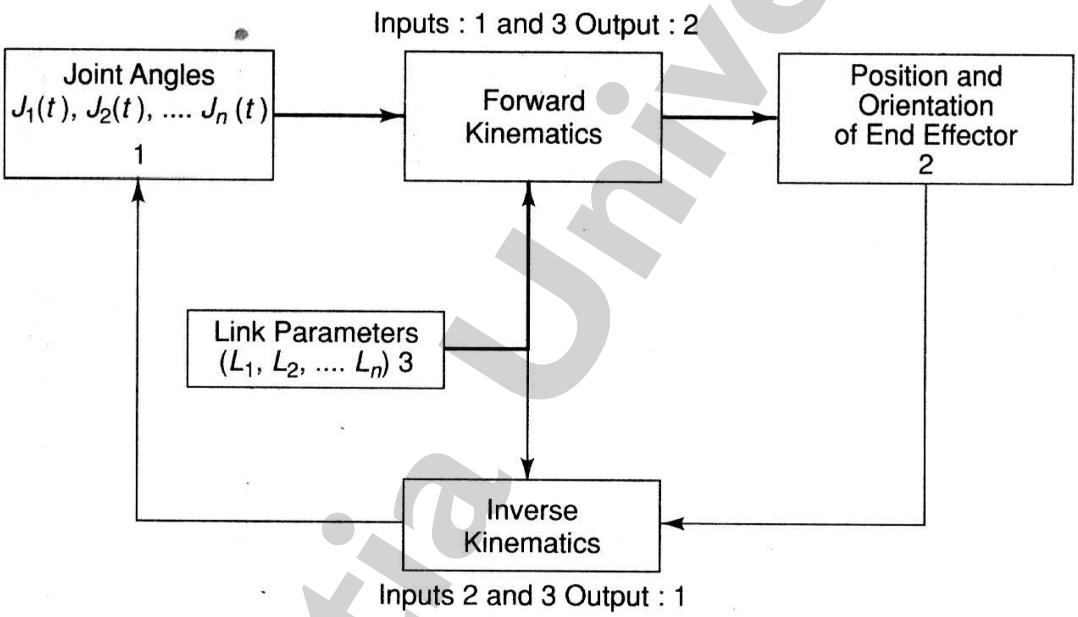


Fig.: Forward and Inverse kinematics scheme

Forward and Reverse kinematics (Transformation) of 3 DOF robot arm:

Forward Transformation

The position and orientation of the end-effector shown in Fig. 2.3 in world space can be determined from the joint angles and the link parameters by the following equations,

$$x_3 = l_1 \cos \theta_1 + l_2 \cos (\theta_1 + \theta_2) + l_3 \cos (\theta_1 + \theta_2 + \theta_3) \quad (2.3)$$

$$y_3 = l_1 \sin \theta_1 + l_2 \sin (\theta_1 + \theta_2) + l_3 \sin (\theta_1 + \theta_2 + \theta_3) \quad (2.4)$$

$$\phi = (\theta_1 + \theta_2 + \theta_3) \quad (2.5)$$

Reverse Transformation

The joint angles can also be determined from the end-effector position (x_3, y_3) and the orientation (ϕ), using reverse transformation in the following way

$$x_2 = x_3 - l_3 \cos \phi \quad (2.6)$$

$$y_2 = y_3 - l_3 \sin \phi \quad (2.7)$$

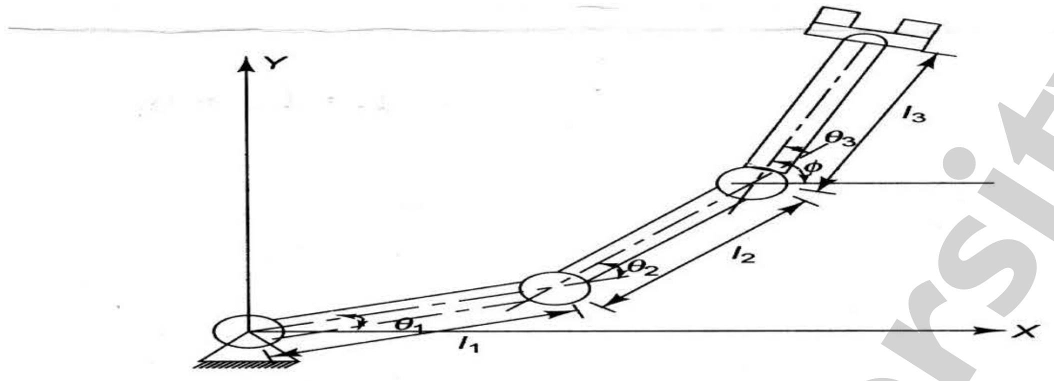


Fig.: 3 DOF 2-D Manipulator

From the given geometry,

$$x_2 = l_1 \cos \theta_1 + l_2 \cos \theta_1 \cos \theta_2 - l_2 \sin \theta_1 \sin \theta_2 \quad (2.8)$$

$$y_2 = l_1 \sin \theta_1 + l_2 \sin \theta_1 \cos \theta_2 + l_2 \cos \theta_1 \sin \theta_2 \quad (2.9)$$

Squaring and adding Eqs (2.8) and (2.9),

$$\cos \theta_2 = \frac{x_2^2 + y_2^2 - l_1^2 - l_2^2}{2 l_1 l_2} \quad (2.10)$$

Substituting the value of θ_2 in Eqs (2.8) and (2.9), we obtain the value of θ_1. Finally, the value of θ_3 can be obtained using the following relation:

$$\theta_3 = \phi - (\theta_1 + \theta_2) \quad (2.11)$$

Forward and Reverse kinematics (Transformation) of 4-DOF robot arm:

A 4-degrees of freedom manipulator in 3-D is illustrated in Fig. 2.4. Joint 1 allows rotation about the z-axis, joint 2 allows rotation about an axis perpendicular to the z-axis, joint 3 is a linear joint and joint 4 allows rotation about an axis parallel to the joint 2 axis.

Let

θ_1 = angle of rotation of joint 1 (base rotation)

θ_2 = angle of rotation of joint 2 (elevation angle)

l = length of the linear joint 3 (extension) (a combination of l_2 and l_3)

θ_4 = angle of rotation of joint 4 (pitch angle)

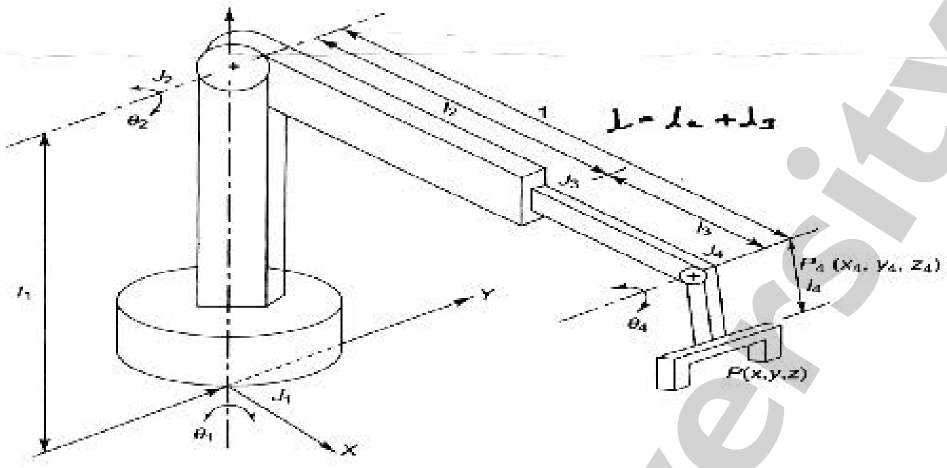


Fig.: 4 DOF 3-D Manipulator

Forward Transformation

The position of the end-effector P in world space is given by

$$x = (l\cos\theta_2 + l_4\cos\theta_4) \times \cos\theta_1 \quad (2.12)$$
$$y = (l\cos\theta_2 + l_4\cos\theta_4) \times \sin\theta_1 \quad (2.13)$$
$$z = l_1 + l\sin\theta_2 + l_4\sin\theta_4 \quad (2.14)$$

Reverse Transformation

If the pitch angle (θ_4) and the world coordinates (x, y, z) of the point P are given, the joint positions can be determined in the following way:

Let the coordinate of the joint 4 be (x_4, y_4, z_4).

Then,

$$x_4 = x - \cos\theta_1 (l_4\cos\theta_4) \quad (2.15)$$
$$y_4 = y - \sin\theta_1 (l_4\cos\theta_4) \quad (2.16)$$
$$z_4 = z - l_4\sin\theta_4$$

Now the values of θ_1, θ_2 and l can be found by

$$\cos\theta_1 = \frac{y_4}{l} \quad (2.17)$$

$$\sin\theta_2 = \frac{z_4 - l_1}{l} \quad (2.18)$$

CONCLUSION: Thus by studying the forward & reverse kinematics for various ROBOT manipulators, we conclude with the demonstration of 2 DOF, 3DOF & 4DOF of ROBOT manipulator.

EXPERIMENT NO.: 3

NAME OF THE EXPERIMENT: Forward Kinematics of PUMA 560

OBJECTIVE:

- To identify the geometric relationship between input and output motion parameters of PUMA 560 robot manipulator.
- Formation of the transformation matrix though which a relationship is established between different links of the manipulator.
- Simulate the robot motion for various inputs of the joint angular value.
- To have a brief idea about the workspace through a 3D graph plot of manipulator position for various inputs.

THEORY:

Programmable Universal Machine for Assembly, more popularly known as PUMA is an industrial robot arm developed by Victor Scheinman at Unimation, in the year 1978. PUMA comes in various makes viz. PUMA 260, PUMA 560, PUMA 761 etc. Figure 2 shows link-frame assignments in the position corresponding to all joint angles equal to zero. Here the frame {0} (not shown) is coincident with frame [1} when is zero. Note also that, for this robot, as for many industrial robots, the joint axes of joints 4, 5, and 6 all intersect at a common point, and this point of intersection coincides with the origin of frames {4}, {5}, and {6}. Furthermore, the joint axes 4, 5, and 6 are mutually orthogonal. This wrist mechanism is illustrated schematically in Fig.4. In this experiment forward kinematics of PUMA 560 is described through a virtual model. The forward kinematics problem is concerned with the relationship between the individual joints of the robot manipulator and the position and orientation of the tool or end effector.

General Terminology in Robotics:

Workspace:

The reachable workspace of a robot's end-effector is the manifold of reachable frames.

Accuracy:

Accuracy refers to a robot's ability to position its wrist end at a desired target point within the work volume, and it is defined in terms of spatial resolution. It depends on the technology and the control increments.

Repeatability:
Repeatability is a statistical term associated with accuracy. If a robot joint moves by the same angle from a certain point a number of times, all with equal environmental conditions, the target is always missed by a large margin. If the same error is repeated, then we say that the repeatability is high and the accuracy is poor.

Safety:
The ability to reduce the human-robot impact force and ensure human safety is a fundamental requirement for human-friendly robots.

Forward Kinematics :
Forward kinematics (FK) mainly deals with constructing a Denavit-Hartenberg (D-H) transformation matrix with Puma's parameters obtained from a D-H parameter table shown below:

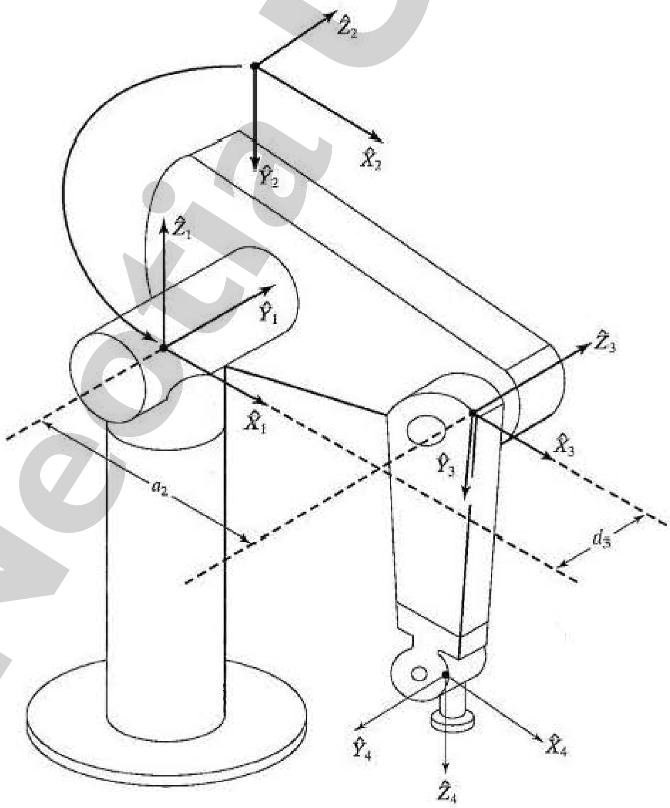


Fig.: Kinematic parameters and frame assignments for the PUMA 560 manipulator.

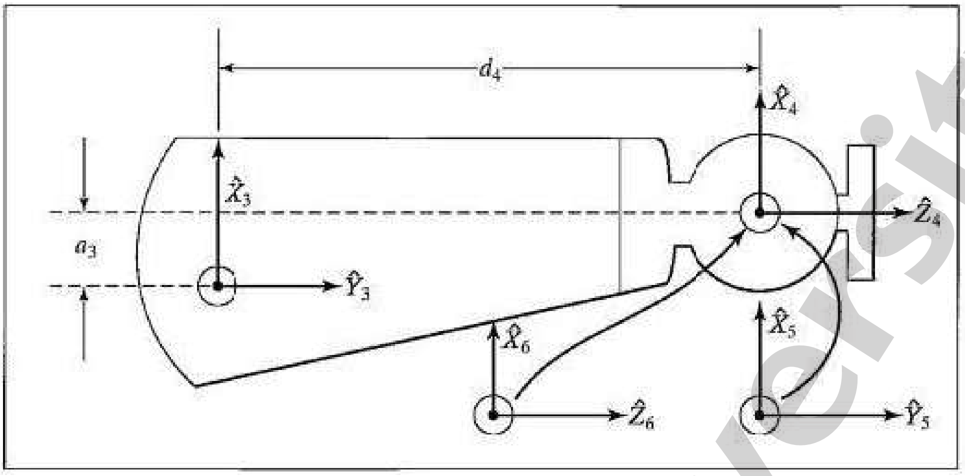


Fig.: Kinematic parameters and frame assignments for the forearm of the PUMA 560 manipulator.

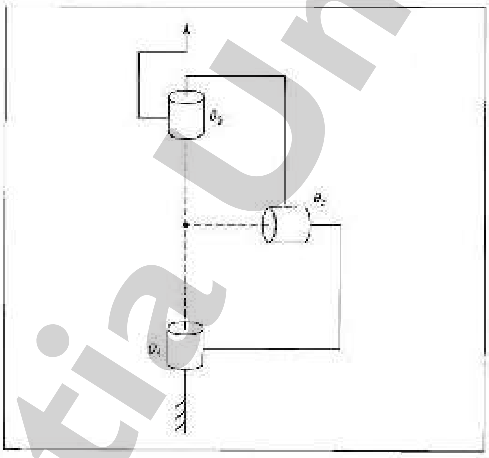


Fig.: Schematic of a 3R wrist in which all three axes intersect at a point and are mutually orthogonal.

Table. Puma 560 D-H parameter table

$i \in k_i$	α_{i-1}	a_{i-1}(M)	d_i(M)	θ_i
1	0	0	0	θ_1
2	-90	0	0	θ_2
3	0	a_2	d_3	θ_3
4	-90	a_3	d_4	θ_4
5	90	0	0	θ_5
6	-90	0	0	θ_6

Transformation matrices of six joints for Puma 560 robot:

$$T_1 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_2) & -\cos(\theta_2) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & a_2 \\ \sin(\theta_3) & \cos(\theta_3) & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_4 = \begin{bmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & a_3 \\ 0 & 0 & 1 & d_4 \\ -\sin(\theta_4) & -\cos(\theta_4) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

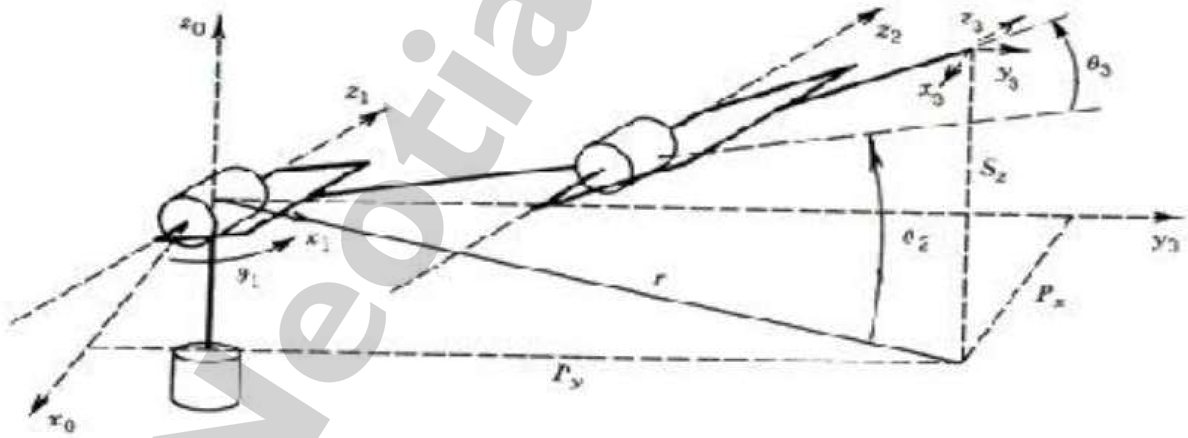
$$T_5 = \begin{bmatrix} \cos(\theta_5) & -\sin(\theta_5) & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin(\theta_5) & \cos(\theta_5) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_6 = \begin{bmatrix} \cos(\theta_6) & -\sin(\theta_6) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin(\theta_6) & -\cos(\theta_6) & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Transformation Matrix $T = T_1 \cdot T_2 \cdot T_3 \cdot T_4 \cdot T_5 \cdot T_6$

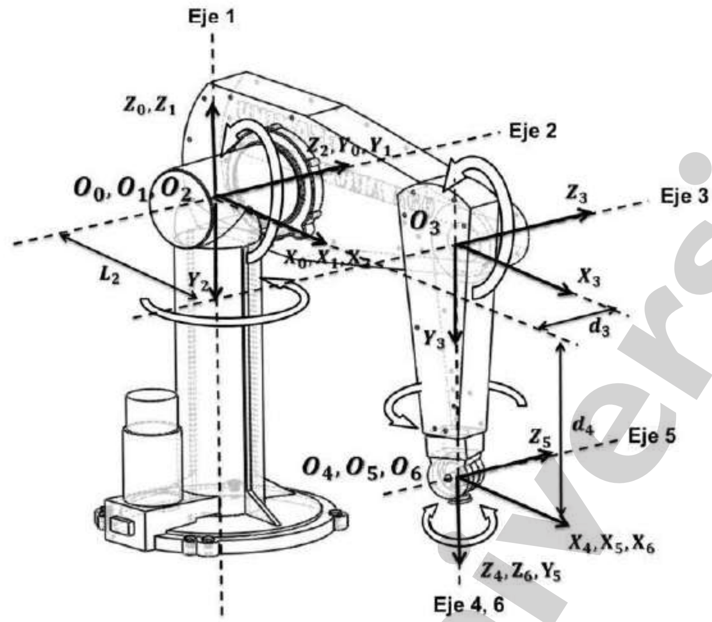
The orientation and position of the end effector with reference to the base coordinate is obtain from the final matri:

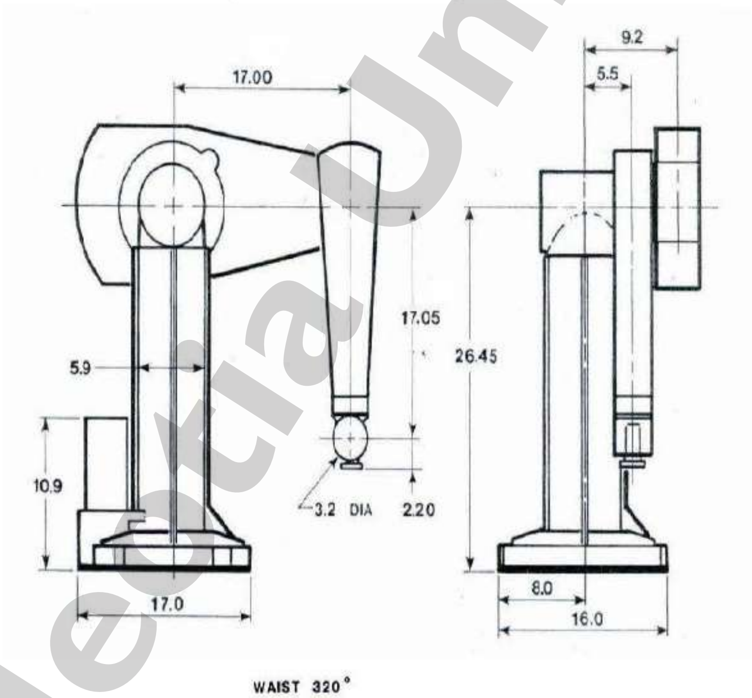
$$T = \begin{bmatrix} n & s & a & p \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

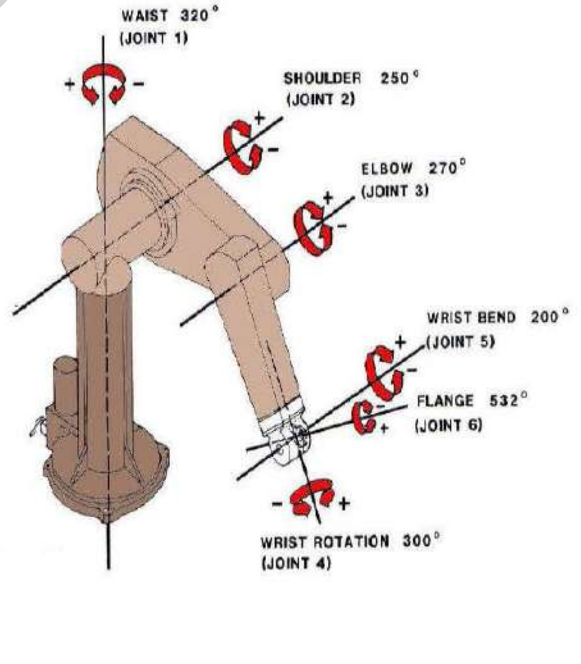
Puma kinematic diagrams:

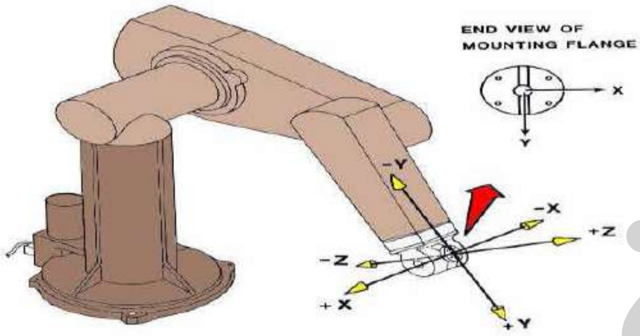


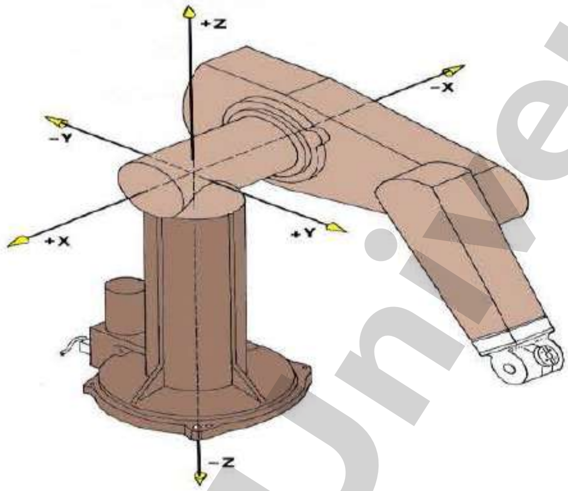
Simplified drawing of first three links of Puma 560 with transformation frames appropriately:


Eje 1
Z_0, Z_1
Eje 2
Z_2, Y_0, Y_1
O_0, O_1, O_2
O_3
Z_3
Eje 3
X_0, X_1, X_2
L_2
Y_2
X_3
d_3
Y_3
d_4
Z_5
Eje 5
O_4, O_5, O_6
X_4, X_5, X_6
Z_4, Z_6, Y_5
Eje 4, 6


17.00
9.2
5.5
17.05
26.45
5.9
10.9
3.2 DIA
2.20
17.0
8.0
16.0


WAIST 320°
(JOINT 1)
SHOULDER 250°
(JOINT 2)
ELBOW 270°
(JOINT 3)
WRIST BEND 200°
(JOINT 5)
FLANGE 532°
(JOINT 6)
WRIST ROTATION 300°
(JOINT 4)





PROCEDURE:

- Insert different values of θ within the joint range as prescribed in theory part and then click ok to get the output orientation and position of the end effector.
- To see the individual movements of the links drag the sliders on the controller panel.
- The Transformation matrix for a particular position and orientation can be obtained either through input panel or via the controller.
- Manipulator position is shown in a 3D graph for every submission of joint values.
- The view can be rotated about a point by keeping the left mouse button pressed and rotating the mouse.
- The view can be translated by keeping the right mouse button pressed and translating the mouse in the desired direction.
- The scroll button or middle mouse button can be used for zooming.

SIMULATOR:

This program simulates a 6 link 3D Puma Robot using the javascript program. The model and its movement of different links are encoded in javascript language. Model With this program the forward kinematics of PUMA 560 is explained, and simultaneously movements of different joints at a time can be seen. Axis coordinates and orientation are setup according to the theory explained earlier. This program does not allow for the specification of angular speed or acceleration of the arms. Click on the screenshot given below to start the simulation.

Kinematics Panel consists of:

ANGLE	RANGE	DOF
θ_1: 320	-160 to +160	Waist Joint
θ_2: 270	-225 to +45	Shoulder Joint
θ_3: 270	-225 to +45	Elbow Joint
θ_4: 280	-110 to +170	Wrist Roll
θ_5: 200	-100 to +100	Wrist Bend
θ_6: 532	-266 to +266	Wrist Swivel

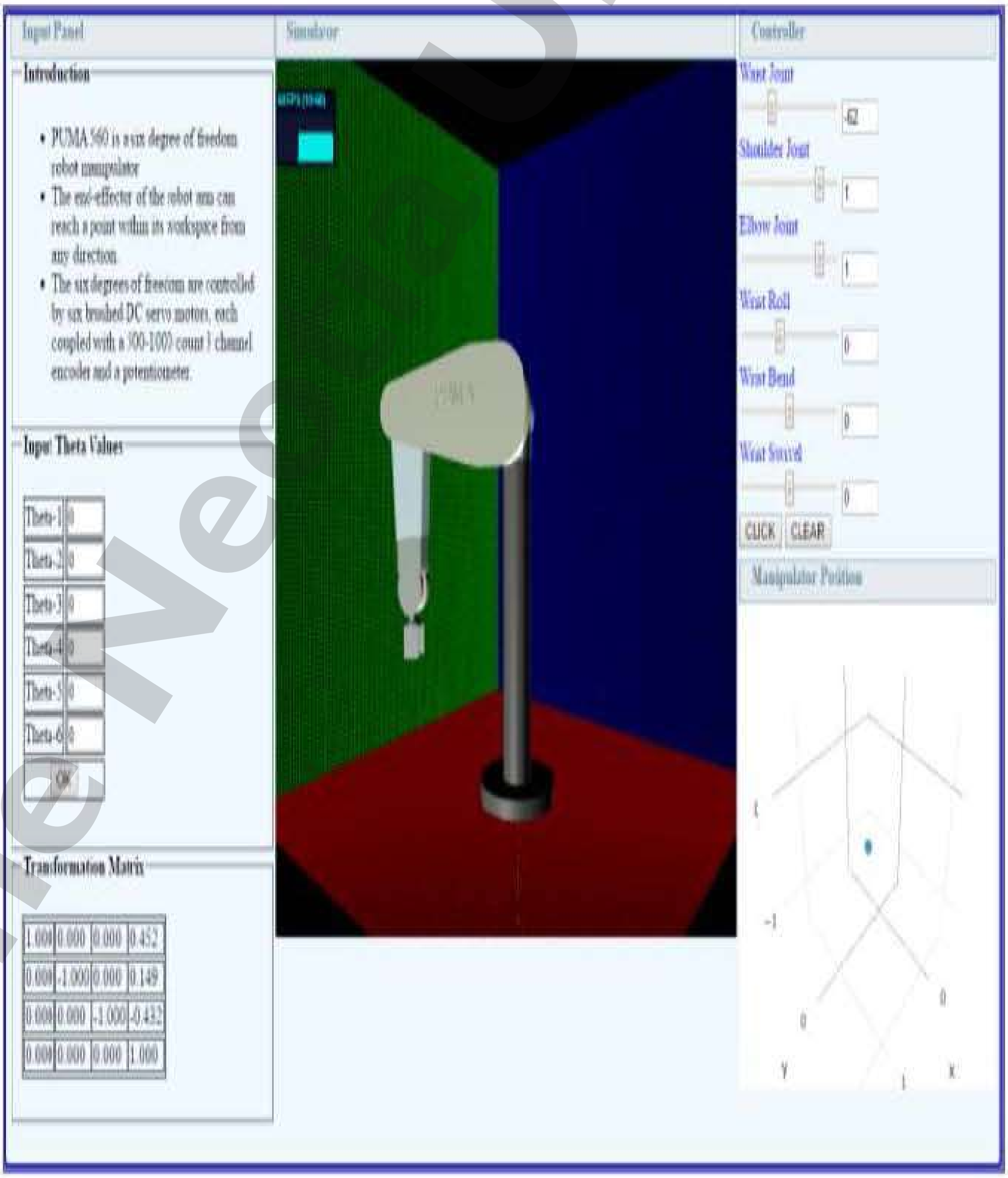


Fig.: Simulator Screen shot

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EXPERIMENT NO.: 4

NAME OF THE EXPERIMENT: Inverse Forward Kinematics of PUMA 560

OBJECTIVE:

- To identify the geometric relationship between input and output motion parameters of PUMA 560 robot manipulator.
- To verify the robot configuration for a particular set of joint solution.
- Simulate the robot motion for various inputs of the manipulator position.

THEORY:

Given the end-effector position and orientation from Forward kinematics problem, the inverse kinematics approach is used to obtain the joint angles. But as stated in the introduction inverse kinematics is more difficult problem than forward kinematics as its include much complexity. The relationship between forward and inverse kinematics is shown in Figure 1. In general there are two main solution techniques for inverse kinematics problem one is analytic approach and other is numerical method. Analytic approach comprises of geometric and algebraic solutions in which joint variables are solved analytically according to given configuration data.

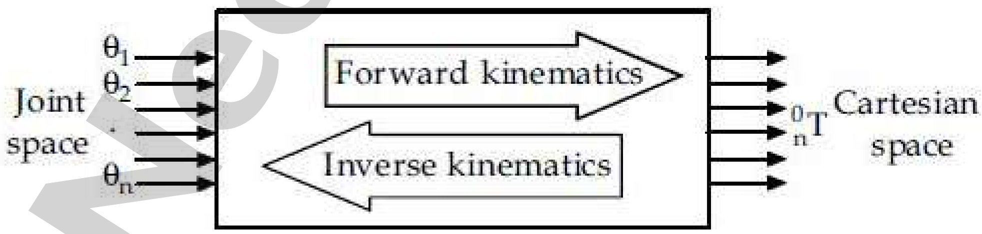


Fig.: The schematic representation of forward and inverse kinematics.

Solving the problem of finding the required joint angles to place the tool frame, {T}, relative to the station frame, {S}, is split into two parts. First, frame transformations are performed to find the wrist frame, {W}, relative to the base frame, {B}, and then the inverse kinematics are used to solve for the joint angles.

Which corresponds to `T_0^4` position

$$p = p_6 - d_6 a = (p_x, p_y, p_z)^T$$

$$\begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} C_1(a_2C_2 + a_3C_{23} + d_4S_{23}) - d_2S_1 \\ S_1(a_2C_2 + a_3C_{23} + d_4S_{23}) + d_2C_1 \\ d_4C_{23} - a_3S_{23} - a_2S_2 \end{bmatrix}$$

Solution for Joint1

$\theta_1^L = \phi - \alpha; \quad \theta_1^R = \pi + \phi + \alpha$

$r = \sqrt{p_x^2 + p_y^2 - d_2^2}; \quad R = \sqrt{p_x^2 + p_y^2}$

$\sin\phi = p_y/R; \quad \cos\phi = p_x/R$

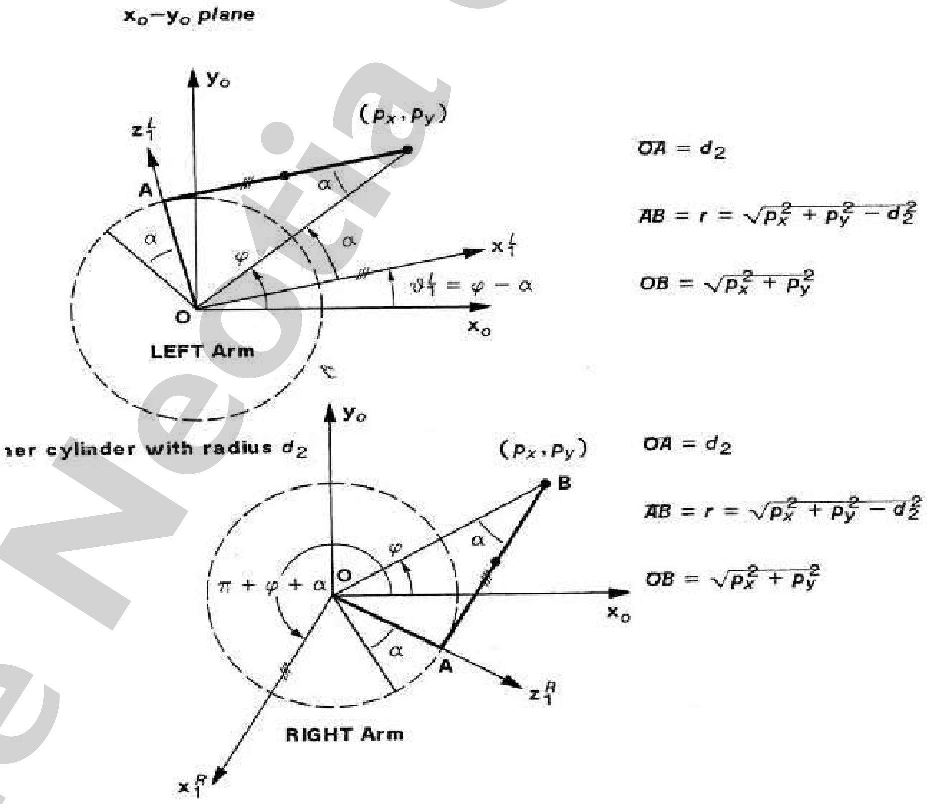
$\sin\alpha = d_2/R; \quad \cos\alpha = r/R$

$\sin\theta_1^L = \sin(\phi - \alpha) = \sin\phi\cos\alpha - \cos\phi\sin\alpha = (p_y r - p_x d_2)/R^2$

$\cos\theta_1^L = \cos(\phi - \alpha) = \cos\phi\cos\alpha + \sin\phi\sin\alpha = (p_x r + p_y d_2)/R^2$

$\sin\theta_1^R = \sin(\pi + \phi + \alpha) = (-p_y r - p_x d_2)/R^2$

$\cos\theta_1^R = \cos(\pi + \phi + \alpha) = (-p_x r + p_y d_2)/R^2$



`sin theta_1 = (-ARM . p_ysqrt(p_x^2 + p_y^2 - d_2^2) - p_xd_2)/(p_x^2 + p_y^2)`

`cos theta_1 = (-ARM . p_ysqrt(p_x^2 + p_y^2 - d_2^2) + p_yd_2)/(p_x^2 + p_y^2)`

`theta = atan2[y/x] = {((0^{\circ} <= theta <= 90^{\circ}; +x and +y), (90^{\circ}<= theta <= 180^{\circ}; -x and +y), (-180^{\circ}<= theta <= -90^{\circ}; -x and -y), (-90^{\circ}<= theta <= 0^{\circ}; +x and -y))`

`theta_1 = atan2[sin theta_1/cos theta_1] = atan2[(- ARM * p_ysqrt(p_x^2 + p_y^2 -d_2^2) - p_xd2)/(- ARM * p_xsqrt(p_x^2 + p_y^2 -d_2^2) + p_yd2)]; -pi<=theta1<=pi`

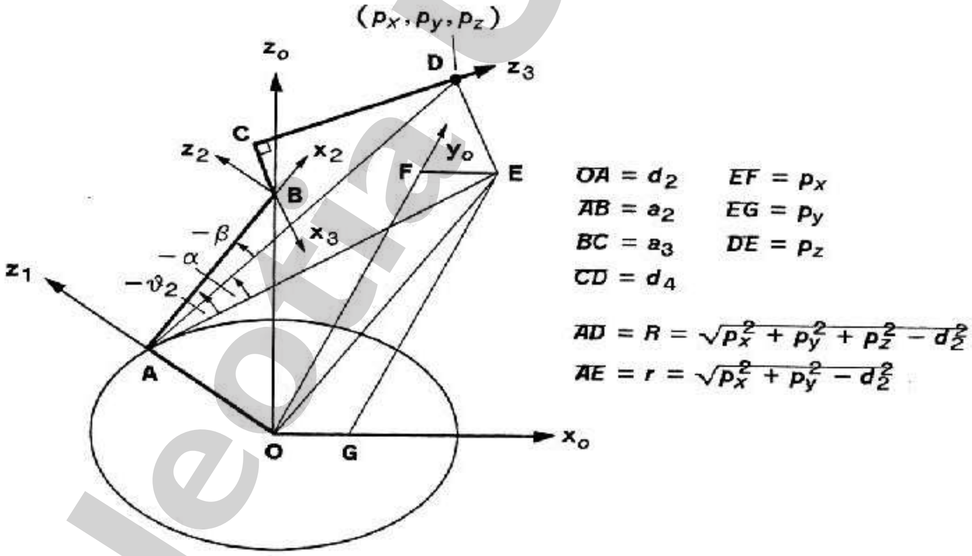
Solution for Joint 2

`R = sqrt(p_x^2 + p_y^2 + p_z^2 -d_2^2) ; r = sqrt(p_x^2 + p_y^2 -d_2^2)`

`sin alpha = -p_z/R = -p/sqrt(p_x^2 + p_y^2 + p_z^2 -d_2^2)`

`cos alpha = -(ARM * r)/R = -(ARM * sqrt(p_x^2 + p_y^2 -d_2^2))/sqrt(p_x^2 + p_y^2 + p_z^2 -d_2^2)`

`cos beta = -(a_2^2 + R^2 - (d_4^2 + a_3^2))/2a_2R = (p_x^2 + p_y^2 + p_z^2 + a_2^2 -d_2^2 - (d_4^2 + a_3^2))/(2a_2sqrt(p_x^2 + p_y^2 + p_z^2 -d_2^2))`



`sin beta = sqrt(1-cos^2 beta)`

`sin theta_2 = sin(alpha + K*beta) = sin alphacos beta + (ARM * ELBOW)cos alphasin beta`

`cos theta_2 = cos(alpha + K*beta) = cos alphacos beta - (ARM * ELBOW)sin alphasin beta`

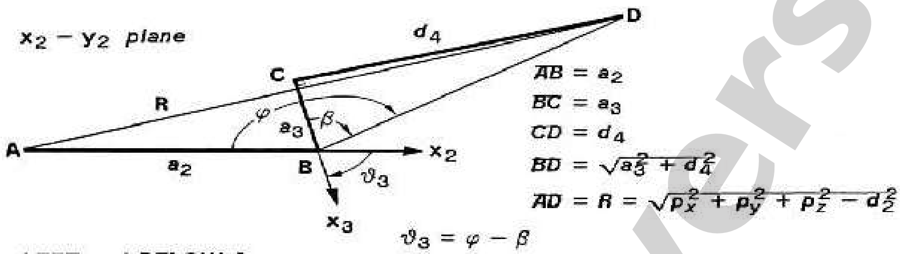
`theta_2 = atan2[sin theta_2/cos theta_2] ; -pi <= theta_2 <= pi`

Solution for Joint 3

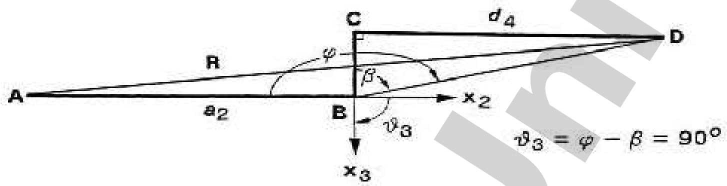
$R = \sqrt{p_x^2 + p_y^2 + p_z^2 - d_2^2}$

$\cos\phi = \frac{a_2^2 + (d_4^2 + a_3^2) - R^2}{2a_2\sqrt{d_4^2 + a_3^2}}$; $\sin\phi = ARM * ELBOW\sqrt{1-\cos^2\phi}$

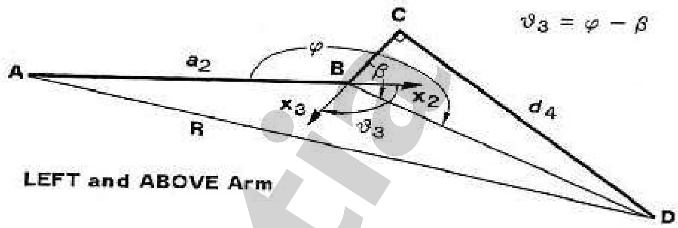
$\sin\beta = \frac{d_4}{\sqrt{d_4^2 + a_3^2}}$; $\cos\beta = \frac{|a_3|}{\sqrt{d_4^2 + a_3^2}}$



LEFT and BELOW Arm



LEFT and BELOW Arm



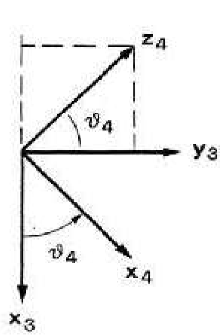
$\theta_3 = \phi - \beta$

$\theta_3 = \operatorname{atan2}[\sin\theta_3/\cos\theta_3]$; $-\pi \le \theta_3 \le \pi$

Solution for Joint 4:

$\sin\vartheta_4 = -(\mathbf{z}_4 \cdot \mathbf{x}_3)$

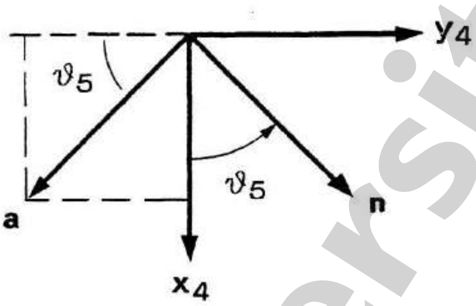
$\cos\vartheta_4 = (\mathbf{z}_4 \cdot \mathbf{y}_3)$



Solution for Joint 5:

$$\sin \vartheta_5 = \mathbf{a} \cdot \mathbf{x}_4$$

$$\cos \vartheta_5 = -(\mathbf{a} \cdot \mathbf{y}_4)$$



`sin theta_6 = n * y_5 ; cos theta_6 = s * y_5`

`theta_6 = atan2[sin theta_6/cos theta_6] ; -pi <= theta_6 <= pi`

` = atan2[((-S_1C_4-C_1C_23S_4)n_x + (C_1C_4-S_1C_23S_4)n_y + (S_4S_23)n_z)/((-S_1C_4-C_1C_23S_4)s_x + (C_1C_4-S_1C_23S_4)s_y + (S_4S_23)s_z)]`

PROCEDURE:

- Insert three position of the end-effector and click OK button to see the joint values.
- Orientation of the manipulator can also be specified.
- The view can be rotated about a point by keeping the left mouse button pressed and rotating the mouse.
- The view can be translated by keeping the right mouse button pressed and translating the mouse in the desired direction.
- The scroll button or middle mouse button can be used for zooming.

SIMULATOR:

This program shows inverse kinematics part of PUMA560. The model and its movement of different links are encoded in javascript language. A default initial position and orientation is given in the input panel and the corresponding output results are shown in the output table. A user can change the position by specifing new values. This program does not allow for the specification of angular speed or acceleration of the arms. Click on the screenshot given below to start the simulation.

Default Ranges of Movements:

ANGLE	RANGE	DOF
`theta_1`: 320	-160 to 160	Waist Joint
`theta_2`: 270	-225 to 45	Shoulder Joint
`theta_3`: 270	-45 to 225	Elbow Joint
`theta_4`: 280	-110 to 170	Wrist Roll
`theta_5`: 200	-100 to 100	Wrist Bend
`theta_6`: 532	-266 to 266	Wrist Swivel

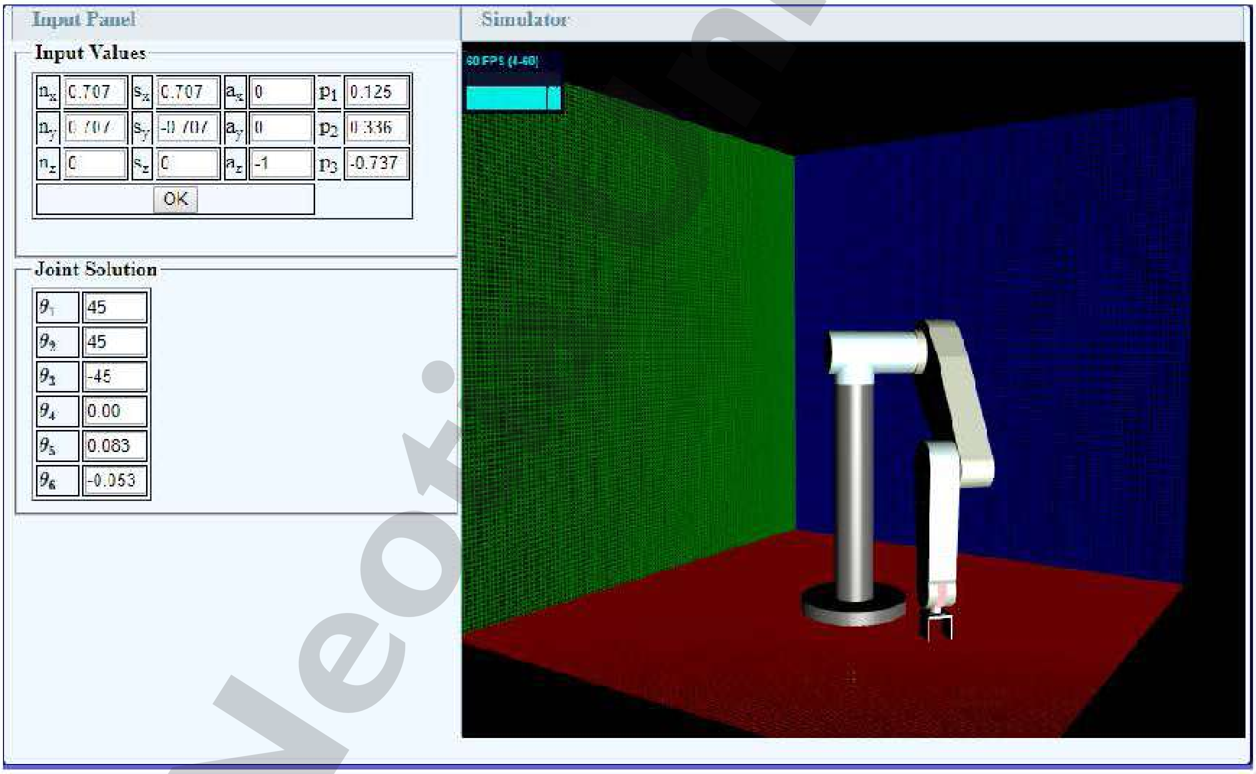


Fig.: Simulator Screen shot

EXPERIMENT NO.: 5

NAME OF THE EXPERIMENT: Two Case Studies of applications in Industry.

(1) Introduction and general considerations in robot applications.
(2) Case study I: Robot application for Welding.
(3) Case study II: Robot application for Spray painting.